

Entanglement as Projection Non-Factorization of One Higher-Dimensional Internal-Phase Object

A three-imaginary-unit operator model with Schmidt-rank, entropy, no-signalling and attosecond-formation consistency checks

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Abstract

We show that if two observed particles are different projections of one parent state in a higher-dimensional or internal-phase Hilbert space, the resulting two-particle wave function is generically non-separable. This provides a natural explanation of entanglement without superluminal signalling. The model is compatible with recent attosecond studies of entanglement formation and with the mathematical, non-traversable interpretation of Einstein-Rosen bridges.

The formalism uses three internal imaginary generators l_1, l_2, l_3 satisfying a Clifford/quaternionic phase algebra. These generators can be represented by ordinary complex matrices, so the construction remains inside standard Hilbert-space operator theory. Entanglement appears when the projection kernel from the parent state to the observed two-particle sector has Schmidt rank greater than one. The entanglement entropy, concurrence and negativity then become standard measures of the non-factorizing projected state. A partial-trace proof shows explicitly that local operations on one projection cannot send controllable signals to the other projection. The interpretation is therefore geometric or operator-theoretic, not superluminal.

1. Purpose and status of the model

This note proposes a mathematical interpretation of entangled particles as distinct observed projections of one parent state living in a larger internal-phase Hilbert space. The word “higher-dimensional” is used operationally: it may mean a literal enlarged configuration space, but it may also mean an internal matrix-phase sector appended to ordinary three-dimensional observation.

The model is not presented as a replacement for standard quantum information theory. Instead, it is a translation framework. The standard measurable object is still the two-particle density matrix ρ_{AB} . Bell tests, reduced density matrices, entropy, concurrence and negativity remain the diagnostic tools. The new claim is that the non-factorization of ρ_{AB} can be generated naturally by a projection map from one coherent parent state rather than by assuming two originally independent particles.

This v3 revision addresses the main reviewer concerns: it connects the projection theorem explicitly to standard entanglement measures, adds a monogamy and Bell-test interpretation, strengthens the no-signalling proof, and clarifies the relation to ER=EPR and holographic entanglement entropy as analogies rather than literal wormhole claims.

2. Internal phase algebra with three imaginary units

Let I_1, I_2, I_3 be internal imaginary generators satisfying

$$\begin{aligned} I_a^2 &= -1, \\ I_a I_b &= -\delta_{ab} 1 + \epsilon_{abc} I_c, \\ I_a I_b + I_b I_a &= -2\delta_{ab} 1. \end{aligned}$$

A concrete representation is obtained with Pauli matrices:

$$I_a = -i \sigma_a.$$

Therefore no non-standard number system is required at the level of observable calculations. The three generators may be represented by ordinary complex matrices acting on an internal two-component or multi-component spinor space.

For any unit vector $n = (n_1, n_2, n_3)$, define an internal imaginary direction

$$I(n) = n_1 I_1 + n_2 I_2 + n_3 I_3, \quad n_1^2 + n_2^2 + n_3^2 = 1.$$

Then

$$I(n)^2 = -1.$$

Thus each coherent internal direction $I(n)$ behaves like a local generalized imaginary unit. Entanglement appears when a single parent state contains more than one coherent internal channel and the observed projection cannot be factorized into one-particle wave functions.

3. Parent Hilbert space and observed two-particle projection

Let the full parent Hilbert space be

$$H_P = H_A \otimes H_B \otimes H_{\text{int}},$$

where H_A and H_B are the two observed one-particle Hilbert spaces and H_{int} is the internal-phase sector. Let the parent state be

$$|\Phi\rangle \in H_P.$$

The observed two-particle wave function is obtained by a projection map

$$P : H_A \otimes H_B \otimes H_{\text{int}} \rightarrow H_A \otimes H_B,$$

or, in coordinate form,

$$\psi_{AB}(x_A, x_B) = \int K(x_A, x_B; \xi) \Phi(x_A, x_B, \xi) d\xi.$$

Here ξ denotes internal coordinates or internal phase labels, and K is the projection kernel. If K factorizes into one-particle kernels and Φ factorizes correspondingly, the observed state may be separable. Otherwise, non-factorization is generic.

3.1 Projection non-factorization theorem

Theorem. Let $|\Phi\rangle$ be a coherent parent state with support on at least two internal channels k . Suppose the projection map has the form

$$|\psi_{AB}\rangle = \sum_k c_k |a_k\rangle \otimes |b_k\rangle,$$

where at least two coefficients c_k are non-zero and the sets $\{|a_k\rangle\}$ and $\{|b_k\rangle\}$ contain at least two linearly independent vectors. Then $|\psi_{AB}\rangle$ is not separable.

Proof. A pure bipartite state is separable if and only if its coefficient matrix has rank one. In the expansion above, the coefficient matrix is a sum of at least two independent rank-one matrices. Under the stated independence assumptions, the resulting matrix has rank greater than one. Equivalently, the Schmidt decomposition of $|\psi_{AB}\rangle$ contains more than one non-zero Schmidt coefficient. Therefore $|\psi_{AB}\rangle$ is entangled.

$$|\psi_{AB}\rangle = \sum_{k=1}^r \lambda_k |u_k\rangle_A |v_k\rangle_B, \quad r > 1.$$

The integer r is the Schmidt rank. In this model, $r > 1$ is not added as a separate postulate. It is produced by the projection of one coherent parent object through more than one internal channel.

3.2 Interpretation

The observed particles A and B need not be interpreted as two independent objects connected by a mysterious action at a distance. They may be read as two boundary projections of one parent state. The correlation is then local in the parent space but nonlocal when only the projected three-dimensional observables are considered.

This does not allow faster-than-light signalling. The measurable theory is still governed by density matrices and partial traces, as shown in Section 6.

4. Entanglement measures

4.1 Reduced density matrix

For a pure two-particle state

$$\rho_{AB} = |\psi_{AB}\rangle\langle\psi_{AB}|,$$

the reduced state of subsystem A is

$$\rho_A = \text{Tr}_B(\rho_{AB}).$$

If $|\psi_{AB}\rangle$ has Schmidt decomposition

$$|\psi_{AB}\rangle = \sum_k \lambda_k |u_k\rangle_A |v_k\rangle_B, \quad \sum_k \lambda_k^2 = 1,$$

then

$$\rho_A = \sum_k \lambda_k^2 |u_k\rangle\langle u_k|.$$

4.2 Entanglement entropy

For a pure two-particle state the von Neumann entropy of the reduced density matrix

$$S(\rho_A) = -\text{Tr}(\rho_A \log_2 \rho_A)$$

quantifies the entanglement. In the projection model $S(\rho_A) > 0$ whenever the parent state Φ has support on more than one coherent internal channel. This directly reproduces the standard signature of entanglement from the geometry of projection.

Using the Schmidt coefficients,

$$S(\rho_A) = -\sum_k \lambda_k^2 \log_2(\lambda_k^2).$$

Therefore

$$S(\rho_A) = 0 \Leftrightarrow \text{one Schmidt coefficient is non-zero} \Leftrightarrow \text{separable state},$$

$$S(\rho_A) > 0 \Leftrightarrow \text{Schmidt rank } r > 1 \Leftrightarrow \text{entangled state}.$$

4.3 Concurrence and negativity

For two qubits, concurrence is a standard entanglement measure. For a pure state with coefficient matrix M ,

$$C = 2 |\det M|.$$

For the Bell-type state

$$|\psi\rangle = (|00\rangle + |11\rangle)/\sqrt{2},$$

one has $C=1$. In the projection model this corresponds to two equal coherent internal projection channels.

For mixed states, negativity is defined by the partial transpose:

$$N(\rho_{AB}) = (\|\rho_{AB}^{\wedge\{T_B\}}\|_1 - 1) / 2.$$

If the projection kernel produces negative eigenvalues under partial transposition, then $N>0$ and the state is entangled. This provides a directly computable test of the model against standard quantum information diagnostics.

4.4 Measure dictionary

Standard measure	Formula	Projection meaning
Schmidt rank	r in $ \psi\rangle = \sum_k \lambda_k u_k\rangle v_k\rangle$	Number of coherent internal projection channels
Entropy	$S(\rho_A) = -\text{Tr}(\rho_A \log_2 \rho_A)$	Information loss under projection to one subsystem
Concurrence	$C=2 \det M $ for pure two-qubit states	Strength of two-channel projection correlation
Negativity	$N=(\ \rho^{\wedge\{T_B\}}\ _1 - 1)/2$	Failure of separability under partial transpose
Mutual information	$I(A:B) = S_A + S_B - S_{AB}$	Total projected correlation

5. Generalized Schrödinger equation with internal phase generators

Let $\Psi(X,t)$ be a wave function on an enlarged configuration space X^M , $M=1,\dots,d$, with an internal matrix phase sector. Choose a local time-phase direction I_0 satisfying $I_0^2 = -1$. Define a covariant derivative

$$D_M = \partial_M + \sum_a B_M^{\wedge a} I_a,$$

and, if an electromagnetic $U(1)$ field is included,

$$D_\mu = \partial_\mu - (q/\hbar) A_\mu I_0 + \sum_a B_\mu^{\wedge a} I_a.$$

The universal Schrödinger-type equation is

$$I_0 \hbar D_t \Psi = [-(\hbar^2/2m)(1/\sqrt{|g|}) D_M (\sqrt{|g|} g^{\wedge\{MN\}} D_N \Psi) + V \Psi].$$

The internal connection $B_M^{\wedge a}$ does not replace the electromagnetic $U(1)$ field. It adds an internal phase-sector connection that can generate non-factorizing projected states.

5.1 Two-particle projection from one parent equation

Suppose Ψ evolves by the parent equation above. The observed two-particle state is

$$\psi_{AB}(x_A, x_B, t) = P\Psi.$$

If the parent evolution preserves coherence between internal channels, the projected ψ_{AB} can acquire Schmidt rank greater than one even when the parent state is geometrically single. This is the operator meaning of “two entangled particles as two projections of one object.”

6. No-signalling theorem

The model must not imply controllable superluminal signalling. This follows from the standard density-matrix formalism.

Let ρ_{AB} be the projected two-particle density matrix. The local state of A is

$$\rho_A = \text{Tr}_B(\rho_{AB}).$$

If observer B applies a local completely positive trace-preserving operation E_B , the joint state becomes

$$\rho_{AB'} = (I_A \otimes E_B)(\rho_{AB}).$$

The reduced state of A is

$$\rho_{A'} = \text{Tr}_B[(I_A \otimes E_B)(\rho_{AB})].$$

For any trace-preserving local operation on B,

$$\rho_{A'} = \rho_A.$$

Thus B cannot change the local statistics available to A by choosing a local operation. The shared projection geometry explains correlations, but it does not create a communication channel.

7. Bell tests, monogamy and standard quantum-information consistency

7.1 Bell correlations

Bell tests constrain local hidden-variable explanations of entanglement. The present model should not be presented as a classical local hidden-variable theory in observed three-dimensional space. Its locality, if any, belongs to the parent internal-phase space. The observed projected state can still violate Bell inequalities exactly as ordinary quantum mechanics predicts.

For the Bell singlet, the standard correlation is

$$E(a,b) = -a \cdot b.$$

The projection interpretation reproduces this by treating the two measurement directions as two boundary readouts of one non-factorizing parent amplitude. The model therefore does not replace Bell-test mathematics; it supplies a geometric/operator origin for the non-factorizing state used in the Bell calculation.

7.2 Monogamy

Entanglement is monogamous: if A is maximally entangled with B, it cannot be independently maximally entangled with a third system C. In the projection model this corresponds to a finite coherent parent channel budget. For three qubits, a standard monogamy inequality is

$$C^2_{\{A|BC\}} \geq C^2_{\{AB\}} + C^2_{\{AC\}}.$$

This can be interpreted as a constraint on how one parent state distributes its internal projection coherence among multiple observed subsystems.

8. Attosecond formation of entanglement

Recent attosecond studies of photoionization examine the time delay and coherence associated with electron ejection and the remaining bound electron. These studies are important for the present model because they suggest that entanglement formation can be treated dynamically during wave-packet splitting rather than as an undefined instantaneous event.

In the projection model, the formation time is the time required for one parent wave packet to split into two distinguishable observed projections while preserving internal coherence. A simplified dynamical reading is

$$\tau_{\text{ent}} \approx \hbar / \Delta E_{\text{int}},$$

where ΔE_{int} is the energy separation between coherent internal channels participating in the projection. If τ_{ent} is in the attosecond range, ΔE_{int} lies naturally in the electron-volt to few-electron-volt scale. This is consistent with atomic photoionization contexts.

The model should not claim that the 232-attosecond value is universal. It is a process-dependent formation time in a specific atomic and laser-driven setting. The conceptual point is narrower: entanglement can have a measurable formation timescale while still preserving no-signalling.

9. Relation to ER=EPR, Einstein-Rosen bridges and holographic entropy

9.1 Non-traversable bridge interpretation

The ER=EPR conjecture suggests a deep relation between entanglement and non-traversable geometric connectivity. The present model is compatible with that intuition, but it does not require literal traversable wormholes. The “bridge” is an operator or projection bridge in the parent Hilbert space.

Recent popular and technical discussions emphasize that Einstein-Rosen bridges should not be treated as ordinary travel tunnels. They may instead express mathematical connectivity, time-structure or non-traversable correlation. This fits the projection view: the connection explains correlation, not transport.

9.2 Holographic entanglement entropy analogy

In holographic duality, the Ryu-Takayanagi formula relates entanglement entropy to a geometric area in the dual bulk:

$$S_A = \text{Area}(\gamma_A) / (4G_N).$$

The present model is not an AdS/CFT derivation. However, it has a parallel structure: an entropy visible in the projected boundary system measures a hidden geometric or internal-phase connection in the parent space. This gives a useful analogy between projection non-factorization and holographic entanglement geometry.

10. Worked two-channel example

Let the parent state have two internal coherent channels:

$$|\Phi\rangle = (|a_0\rangle|b_0\rangle|\xi_0\rangle + |a_1\rangle|b_1\rangle|\xi_1\rangle) / \sqrt{2}.$$

Let the projection erase the internal label while preserving coherence:

$$P|\xi_0\rangle = 1, \quad P|\xi_1\rangle = 1.$$

Then

$$|\psi_{AB}\rangle = (|a_0\rangle|b_0\rangle + |a_1\rangle|b_1\rangle) / \sqrt{2}.$$

If $|a_0\rangle$, $|a_1\rangle$ and $|b_0\rangle$, $|b_1\rangle$ are orthonormal pairs, this is a maximally entangled state. The reduced density matrix is

$$\rho_A = 1/2 |a_0\rangle\langle a_0| + 1/2 |a_1\rangle\langle a_1|,$$

so

$$S(\rho_A) = 1 \text{ bit}.$$

Thus entanglement entropy arises directly from the projection of one coherent parent state through two internal channels.

11. Predictions and falsifiable directions

The model suggests several testable or at least data-facing directions:

- Entanglement formation should be describable as a finite coherence-transfer process in ultrafast wave-packet splitting experiments.
- The effective formation time should depend on the internal energy splitting ΔE_{int} and on the strength of the projection coupling.
- Higher-dimensional or internal-phase projection models must reproduce all standard entanglement measures of the projected density matrix.
- No-signalling must remain exact after tracing over unobserved internal channels.
- For multi-party systems, monogamy inequalities should translate into constraints on the available coherent internal projection channels.

12. Limitations

This is an effective mathematical model, not a completed derivation from the Standard Model. The internal phase connection B_M^a is introduced as an operator structure whose physical origin must be constrained by experiment. The model must be judged by whether it reproduces standard quantum information predictions while adding useful intuition or new calculational tools.

The ER=EPR and holographic entropy sections should be read as structural analogies. They do not claim the existence of traversable wormholes or macroscopic tunnels connecting entangled laboratory particles.

13. Conclusion

The projection model gives a compact explanation of entanglement: two observed particles can be non-separable because they are two projections of one coherent parent state in a larger internal-phase Hilbert space. The three-imaginary-unit algebra provides a clean operator language for internal phases, while the projection theorem shows why non-factorization is generic.

The model reproduces the standard mathematical signatures of entanglement: Schmidt rank greater than one, positive von Neumann entropy, nonzero concurrence or negativity, Bell-type

correlations and no-signalling through partial trace. Attosecond entanglement-formation studies support the idea that entanglement can be treated as a finite dynamical formation process rather than as magic or instantaneous action. The ER/EPR and holographic comparisons suggest a broader interpretation: entanglement may be the projected shadow of hidden connectivity, not a superluminal force.

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Author note. This document is a research note for discussion. It proposes an effective projection model and should be tested against the full body of quantum-information and quantum-foundation literature before journal submission.